**PURPOSE:** To describe the new sediment transport formulation in GSSHA and demonstrate the improved capability in predicting storm-total sediment runoff using a research-quality data set.

**BACKGROUND:** GSSHA simulates overland soil erosion and outputs erosion and deposition for any size class of particles smaller than gravel with specific gravities greater than 1.0. The model first calculates particle detachment by raindrops and surface runoff, and then calculates the transport capacity of surface runoff using one of three user-selected transport equations. The actual sediment transport is determined by comparing the amount of detached soil and the transport capacity of surface runoff. Depending on particle size, soil transported to channels is treated either as wash load or bed load, with sizes less than the user specified value of sand treated as wash load and larger sizes treated as bed load. The original GSSHA sediment transport formulation was hard-coded to simulate only three size classes of sediment (sand, silt, and clay), each with a specific gravity of silicate minerals (S=2.65). The new GSSHA erosion routines have been generalized to allow simulation of arbitrary size classes smaller than gravels, each with unique specific gravity.

Ogden and Heilig (2001) performed a detailed investigation of the CASC2D erosion formulation (Johnson 1997, 2000) and found that it greatly over-predicts storm-total sediment yield for events that are considerably larger than the event used to calibrate the model. They attributed this to the lack of detachment limits in the original CASC2D formulation, which assumes that transport capacity of overland flow is always satisfied by erosion. Kalin and Hantush (2006) compared the Kineros-2 model with the CASC2D erosion formulation included in prior versions of GSSHA. Their study revealed that while the hydrologic components of GSSHA out-perform Kineros-2, the lack of detachment limits in the CASC2D erosion formulation was problematic. This re-formulation of the erosion routines in GSSHA was performed partly to address this issue. Kalin and Hantush (2006) also suggested that the raster channel representation in GSSHA distorted channel lengths and slopes, requiring the use of artificially-low values of Manning’s n. To address this issue the channel representation in GSSHA was changed from raster to vector. The effect of this change on representativeness of the Manning’s n values used in the enhanced GSSHA formulation is examined as well.

**SEDIMENT TRANSPORT FORMULATION:** To address the many concerns identified with the original sediment transport formulation developed in CASC2D, the sediment erosion and transport formulation in GSSHA was redeveloped to function in the following general manner.
1. Soils on the overland flow plane are comprised of a user specified number of sediments with specified diameters, specific gravities, and fractional distribution in each overland flow cell.

2. GSSHA keeps sediment particles in three separate bins, parent soil material, suspended sediment, and deposited materials.

3. During a rainfall event, sediment particles are detached by rainfall and overland flow. Deposited materials are eroded first, if available. If there is no deposited layer, or the amount of deposited materials is less than the detachment forces, then parent soil materials are eroded. This detachment process sets an upper limit on the amount of sediments that can be transported from a given cell.

4. Next, the transport capacity is calculated using the user specified method. If the transport capacity is greater than the available materials, then all of the suspended materials are transported to downstream cells by advection, using a two-dimensional sequential explicit method.

5. If the transport capacity cannot accommodate all of the suspended materials, then some materials may be deposited according to the calculated trap efficiency. Materials deposited in cells are added to the deposition layer in that cell and are eroded first in subsequent time intervals.

6. Materials not deposited or transported remain in suspension, and may be transported or deposited in subsequent time steps.

7. Upon reaching a stream cell, suspended, and deposited materials, in an overland flow cell containing a stream node are added to the stream node.

8. Once in the stream network, particles are attributed as either wash load or bed load depending on whether the particles’ diameters are larger or smaller than the user-specified value of sand.
   a. Particles sand size and larger are transported as bed load using Yang’s method.
   b. Particles smaller than sand size are considered wash load and are transported using the advection-dispersion equation. Particles considered wash load do not settle in fluvial stream links or structures but may settle in reservoirs depending on their calculated fall rate and reservoir properties.

9. Sediment transport is now compatible with long-term simulations. If long-term simulations are specified by the user, changes to the parent and depositional layer on the overland flow plane and channel cross section in one event are carried forward to the next event. If the user chooses, changes in overland flow cell elevation due to erosion or deposition can also be carried forward to the next event. To save computational time, any suspended sediments remaining in suspension at the end of a specified event are deposited into the overland flow cell containing the suspended sediments and sediment computations cease until the next specified event.
Sediment Detachment. Soil detachment can occur due to rainfall and overland flow. Total detachment is comprised of the sum of rainfall and overland flow detachment.

Detachment by raindrops. Detachment by raindrops is considered to be a function of rainfall momentum, which is also a function of rainfall intensity. Raindrop detachment also takes into account additional factors including surface water cushion, ground cover, and plant interception (Foster, 1982; Wicks and Bathurst, 1996):

\[ D_R = K_i C_w C_G C_i M_R \]  

where:

- \( D_R \) = detachment capacity rate (kg m\(^{-2}\) s\(^{-1}\))
- \( K_i \) = soil erodibility factor for detachment by raindrop impact (J\(^{-1}\))
- \( C_w \) = water depth correction factor
- \( C_G \) = canopy cover factor
- \( C_i \) = a cover-management factor
- \( M_R \) = moment squared for rainfall ((kg m s\(^{-1}\))^2 m\(^{-2}\) s\(^{-1}\))

Table 1 lists a set of values of the soil erodibility factor for different soils. These values were calculated from experimental data by various researchers, as listed in Table 1.

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Clay</th>
<th>Silty Clay</th>
<th>Silty Clay Loam</th>
<th>Silt</th>
<th>Silt Loam</th>
<th>Loam</th>
<th>Sandy Loam</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyer and Harmon(1984)</td>
<td>19.0</td>
<td>18.2</td>
<td>16.2</td>
<td>29.8</td>
<td>39.8</td>
<td>28.2</td>
<td>32.0</td>
<td></td>
</tr>
<tr>
<td>Morgan (1985)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>Bradford et al. (1987a,b)</td>
<td>73.5</td>
<td>22.2</td>
<td>25.7</td>
<td>37.6</td>
<td>34.4</td>
<td>62.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verhaegen (1987)</td>
<td></td>
<td></td>
<td></td>
<td>24.7</td>
<td>23.4</td>
<td>30.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The relationship between the momentum squared and intensity of rainfall is highly nonlinear. It is expressed as a power function:

\[ M_R = \alpha I^\beta \]  

Where \( I \) is rainfall intensity (mm h\(^{-1}\)), and \( \alpha \) and \( \beta \) are empirical coefficients related to rainfall intensity. Standard values for these parameters are shown in Table 2.
Table 2. Parameters for the relationship between momentum squared and rainfall intensity $I$, after Wicks and Bathurst (1996).

<table>
<thead>
<tr>
<th>Rainfall Intensity (mm h$^{-1}$)</th>
<th>$\alpha$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ $I$ ≤ 10</td>
<td>2.69*10$^{-8}$</td>
<td>1.6896</td>
</tr>
<tr>
<td>10 ≤ $I$ ≤ 50</td>
<td>3.75*10$^{-8}$</td>
<td>1.5545</td>
</tr>
<tr>
<td>50 ≤ $I$ ≤ 100</td>
<td>6.12*10$^{-8}$</td>
<td>1.4242</td>
</tr>
<tr>
<td>100 ≤ $I$ ≤ 250</td>
<td>11.75*10$^{-8}$</td>
<td>1.2821</td>
</tr>
</tbody>
</table>

The calculation of water depth correction factor is based on the assumption that surface water depths greater than a critical depth protect the soil from raindrop impact erosion. The expression for the correction factor, $C_w$, is:

$$C_w(h,d) = \begin{cases} \exp(1-h/D_m) & \text{if } h > D_m \\ 1 & \text{if } h \leq D_m \end{cases}$$

where $h = \text{water depth (m)}$, and $D_m = \text{median raindrop diameter (m)}$.

The median raindrop diameter is determined from the Laws and Parsons (1943) equation:

$$D_m = 0.00124 I^{0.182}$$

**Detachment by surface runoff.** Surface runoff detaches soil particles by exerting a shear stress that breaks the bonds between particles. Erosion in rills is lumped and described as gross rill erosion. Within a grid cell, rill erosion and flow are assumed to be uniformly distributed. The detachment capacity rate by surface runoff has the form:

$$D_c = a(\tau - \tau_{cr})^b (1-G/T_c)$$

where:

- $D_c = \text{detachment capacity rate (kg m}^{-2} \cdot \text{s}^{-1})$
- $a$ and $b$ are empirical coefficients
- $\tau = \text{the flow shear stress (Pa)}$
- $\tau_{cr} = \text{the critical shear stress}$
- $G = \text{the sediment load (kg m}^{-2} \cdot \text{s}^{-1})$
- $T_c = \text{the sediment transport capacity of surface runoff (kg m}^{-2} \cdot \text{s}^{-1})$

In the model, it is assumed that detachment is linearly proportional to the excess shear stress, and $b$ is taken as 1 (Flanagan and Nearing, 1995).
**Sediment Transport Capacity of Surface Runoff.** Two alternative methods have been added to the GSSHA formulation to calculate the sediment transport capacity of surface runoff in addition to the original Kilinc-Richardson (1973) equation.

The Kilinc and Richardson (1973) sediment transport equation, as modified by Julien (1995), and Ogden and Heilig (2001) is used in the current GSSHA code. Sediment discharge by means of overland flow is a function of the hydraulic properties of the flow, the physical properties of the soil, and surface characteristics as given by:

\[
q_s = 25500 q^{2.035} S_f^{1.664} \frac{K}{0.15}
\]  \hspace{1cm} (6)

where:

- \(q_s\) = sediment unit discharge (ton m\(^{-1}\) sec\(^{-1}\))
- \(q\) = unit discharge (m\(^2\) sec\(^{-1}\))
- \(S_f\) = friction slope (-)

The factor \(K\) in Equation 6 is a combined factor that describes soil erodibility, land-use and land-cover characteristics. It can be conceptualized to consist of multiplicative terms similar to the USLE soil erodibility factor (0-1), soil cropping factor (0-1) and conservation factor (0-1) in the development by Julien (1995). The use of one factor \(K\) represents a departure from Julien (1995), who used all three factors from the Universal Soil Loss Equation (USLE). This departure is justified by questions regarding the applicability of the USLE terms at the time scale of individual storm events.

The Engelund-Hansen (1967) equation can be used to calculate sediment transport for each soil size and the resulting total transport is calculated by multiplying the proportion of the size in the parent material by the calculated rate

\[
G_i = KF_i \frac{0.05BV^{2}h^{3/2}S^{3/2}}{(s-1)^{2}D_i\sqrt{g}}
\]  \hspace{1cm} (7)

where:

- \(G_i\) = the volumetric sediment transport rate of \(i\)-th size fraction
- \(K\) = the calibration coefficient (= 1 for standard equation)
- \(F_i\) = the proportion of \(i\)-th fraction in the parent material or deposited layer
- \(B\) = the width of flow
- \(V\) = the mean water velocity
- \(h\) = the flow depth
- \(S\) = the water surface slope
- \(s = \rho_s/\rho\) is the specific gravity of \(i\)-th fraction
- \(g\) = the gravitational acceleration
\[ \rho_s = \text{the sediment density} \]
\[ \rho = \text{the water density} \]
\[ D_i = \text{the mean size of } i\text{-th fraction} \]

The factor 0.05 in Equation 7 was determined using empirical data. The suggested applicability of the Engelund-Hansen equation is for \( \sqrt{D_{25}/D_{50}} < 1.6 \) (\( D_N \) is the grain size for which \( N \) percent of sediment is finer by weight) and for sand-size sediments coarser than 0.15 mm.

Several excess-shear methods that are similar to those used in the WEPP (Flanagan and Nearing, 1995) model can also be chosen to calculate the transport capacity of surface runoff. The general expression has the form:

\[ T_c = a (\omega - \omega_c)^b \]  (8)

where:

\[ a = \text{the transport coefficient} \]
\[ b = \text{the exponent} \]
\[ \omega = \text{shear stress, steam power, or unit stream power} \]
\[ \omega_c = \text{a critical shear stress, stream power, or unit stream power} \]

A comparison of the three sediment transport relations denoted by Equations 6, 7, and 8 was performed by running the GSSHA model with each equation. Surprisingly, the result of this comparison showed that for sediments with \( S=2.65 \), there is very little difference between them. Therefore, the user is advised to use the Kilinc-Richardson (1973) method because it has the smallest number of parameters. However, for simulations involving sediments with specific gravities different from 2.65, the use of the Engelund-Hansen (1967) equation is required.

**Sediment Transport in Channels with Breakpoint Cross-sections.** The GSSHA model employs the unit stream power method of Yang (1973) for routing sand-size total-load in stream channels. Unit stream power is defined as the product of the average flow velocity \( U \) and the channel slope \( S \). Soil erosion in channels is considered transport-limited.

In the GSSHA model, the routing formulation for sand-size sediments works for both trapezoidal and natural channel cross-sections. The degradation of trapezoidal channels is assumed to occur uniformly from the bottom of the channel. Degradation can continue and bed load is transported at the rate calculated with the Yang (1973) method until the user-specified maximum degradation is reached. Deposition is assumed to cover the entire bottom width of the channel, whether in the previously eroded bottom or in the trapezoidal section.

In the case of natural channels with breakpoint cross-section definitions, GSSHA interpolates from the known X-Y pairs to get 60 X-Y pairs for each breakpoint cross-section. Channel geometry is updated after each rainfall event that generates surface runoff. GSSHA tracks the highest water level in the channel during a rain event and uses it as a point below which channel erosion or deposition is applied as a linear function of depth, increasing from zero at the
maximum water surface elevation to a maximum at the channel bottom. To take the gravitational force into consideration, deposition is assumed to be proportional to the square of water depth, starting from the recorded maximum water surface elevation. The algorithm implemented in GSSHA calculates the elevation change of all points below the highest water level so that the mass of sediment is conserved.

Particles with sizes smaller than the user-specified value of sand are assumed to be in suspension upon entering the channel, and are transported as wash load using the advection-diffusion method (Downer and Byrd, 2007). Routing of suspended fines is a natural extension of the explicit diffusion wave channel routing method. Suspended fine sediments are routed as concentrations.

**Example Application on Goodwin Creek.** The USDA-ARS Goodwin Creek Experimental Watershed (GCEW) data set was used to test the modified GSSHA model. The GCEW is a 21.1 km² watershed near Batesville, Mississippi. The USDA-ARS has collected rainfall, runoff, and suspended sediment data in the watershed since 1986 (Figure 1). The goals of numerical tests included (1) testing of the hydrological component after the new representation of channel network was introduced; and (2) testing of the new soil erosion routines, especially their ability to predict sediment yield under heavy rainfall events, where Ogden and Heilig (2001) found that the original CASC2D erosion formulation greatly overestimated erosion during events that are significantly larger than the calibration event.

The hydrologic and sediment measurements from the USDA-ARS National Sedimentation laboratory Goodwin Creek Experimental Watershed (GCEW), which were used to test the soil erosion and sediment routine in CASC2D by Ogden and Heilig (2001), were used to test the sediment transport and other new features affecting sediment transport in the GSSHA model. The new features required re-calibration and validation of the model. The Goodwin Creek data
set from May 22, 1982 to August 30, 1982 (Day of year 142 to 243) was used to calibrate the model and data from 1999 were used to verify the hydrologic response of the model (Downer, 2008). Hydrologic calibration included identification of a suitable value for the Manning roughness coefficient of the channel network. In the calibration process, the channel roughness coefficient increased from 0.028 to 0.035, reflecting the shorter average length of the links (Downer, 2008). The value of 0.035 is a more physically realistic value for the GCEW channels. The number of links in the Goodwin Creek channel network was increased from 18 to 29 to more accurately reflect the observed drainage network.

![Figure 2. Observed and simulated discharge from GCEW calibration period (after Downer, 2008)](image)

Calibration results are shown in Figure 2. Mean Absolute Error (MAE) for event peak discharge and volume for this period was 21 and 27 percent, respectively (Downer, 2008). Calibration of the erosion-related parameters focused on erodibility factor $K$ for nine separate land use/soil complexes, and proceeded using the following approach:

1. The Kilinc-Richardson (1973) equation was used to predict the transport capacity of overland flow.
2. Channel suspended load was simulated using advection-dispersion as in the original GSSHA formulation, however in the new non-orthogonal channel network.

3. The value of the raindrop erosivity coefficient $K_I$ in Equation 1 was assumed to be equal to 60 J-1.

4. The soil erosivity values for each unique soil texture/land-cover class was identified using the Shuffled Complex Evolution method (Duan et al. 1992).

5. The coefficient $a$ used for surface detachment in Equation 5 was $2 \times 10^{-5}$.

**Sediment modeling results.** Figure 3 shows the sedographs computed by GSSHA during the calibration period. Sedograph numbers in Figure 3 correspond to the hydrograph numbers in Figure 2. The sedograph for the small non-numbered event between hydrographs 2 and 3 is included in Figure 3. Table 3 lists the observed and simulated sediment runoff volumes, and error in sediment volume for the events shown in Figure 2. One additional event is included in the analysis in Table 3. The hydrograph from this small event is not pictured (NP) in Figure 2.

<table>
<thead>
<tr>
<th>Event</th>
<th>Observed Peak Discharge (m$^3$ s$^{-1}$)</th>
<th>Observed Discharge Volume (1000 m$^3$)</th>
<th>Observed Sediment Volume (m$^3$)</th>
<th>Simulated Sediment Volume (m$^3$)</th>
<th>Error (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.8</td>
<td>224.3</td>
<td>624</td>
<td>369</td>
<td>-41</td>
</tr>
<tr>
<td>2</td>
<td>12.9</td>
<td>168.6</td>
<td>500</td>
<td>184</td>
<td>-63</td>
</tr>
<tr>
<td>NP</td>
<td>0.98</td>
<td>19.6</td>
<td>3</td>
<td>3.8</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>15.7</td>
<td>106.6</td>
<td>240</td>
<td>181</td>
<td>-24</td>
</tr>
<tr>
<td>4</td>
<td>6.1</td>
<td>62.2</td>
<td>30</td>
<td>96</td>
<td>220</td>
</tr>
<tr>
<td>5</td>
<td>18.3</td>
<td>192.0</td>
<td>256</td>
<td>269</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>9.9</td>
<td>77.9</td>
<td>87</td>
<td>23</td>
<td>-73</td>
</tr>
<tr>
<td>7</td>
<td>5.7</td>
<td>48.8</td>
<td>35</td>
<td>28</td>
<td>-20</td>
</tr>
<tr>
<td>8</td>
<td>38</td>
<td>375.9</td>
<td>640</td>
<td>640</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>2415</td>
<td>1797</td>
<td>-26</td>
</tr>
</tbody>
</table>
Figure 3. GSSHA sediment discharge predictions and observations from Goodwin Creek.
As seen in Table 3, during the calibration period, the mean absolute error in predicted sediment volume is 52 percent and total error for the simulation is -26 percent.

The model was validated against two large events that occurred in 1983. These events produced peak discharges of 71 m$^3$ s$^{-1}$ and 148 m$^3$ s$^{-1}$, approximately 2 and 4 times larger, respectively, than any event in the calibration period. All calibrated parameter values remained the same. The initial moisture was adjusted to better match the discharge from the first event. The model results are shown in Table 4.

<table>
<thead>
<tr>
<th>Event</th>
<th>Observed Peak Discharge (m$^3$ s$^{-1}$)</th>
<th>Observed Discharge Volume (1000 m$^3$)</th>
<th>Observed Sediment Volume (m$^3$)</th>
<th>Simulated Sediment Volume (m$^3$)</th>
<th>Error (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71</td>
<td>788</td>
<td>1216</td>
<td>1254</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>148</td>
<td>2996</td>
<td>5066</td>
<td>2664</td>
<td>-47</td>
</tr>
</tbody>
</table>

Comparison to prior results. One of the many concerns expressed about the previous formulation of the sediment transport in CASCD by Ogden and Helig (2001) was that small events were poorly simulated. This was thought to be due to several reasons, including a lack of constraint on the transport capacity, errors in initial soil moisture, and errors in hydrologic predictions. Figure 4 shows the percent error in sediment volume for all calibration and verification events. This simple analysis of the errors in predicting sediment volume and the size of the storm event, defined by the peak discharge, shows that while there is a general improvement in prediction error with larger events, the model is able to simulate events of all sizes, including the small events included in the calibration period as shown in Figure 3, third panel. This is in sharp contrast to earlier findings by Ogden and Helig (2001) who found that CASC2D was unable to approximately simulate the sediment volume from smaller events.
Another concern of Ogden and Helig (2001) was propagation of errors - that errors in simulating discharge would lead to larger errors in predicting sediment discharge. While this is true in a general sense, the error in predicting discharge is less than half of the error in predicting sediment discharge. Since most errors in sediment volume fall around the MAE 50 percent line regardless of the accuracy of the discharge prediction, there is no general trend between hydrologic prediction error and sediment discharge error (Figure 5). Factors controlling sediment discharge predictions in the model extend beyond the ability of the model to simulate discharge in the stream channel. One data point that stands out in both Figures 4 and 5 is Event 5. Although both the discharge and sediment discharge was very similar to those of Event 7 of the calibration period, the GSSHA model severely overestimated the sediment discharge for the event even though the prediction of discharge was just as accurate as that for Event 7. The reason for the anomaly is unknown.
The last concern of Ogden and Helig (2001) addressed in this document is that the sediment transport routines in CASC2D grossly overestimated sediment discharge volumes from large events. This was thought to be caused by the method always satisfying the transport capacity regardless of erosional supply of sediments. The verification events listed in Table 5 indicate that the model as currently formulated can reproduce the sediment runoff from very large events, even when no such events are included in the calibration period. And as can be seen in Figure 6, the model is capable of reproducing sediment volumes from events that range in peak discharge and sediment volume more than three orders of magnitude at the GCEW.
Conclusion. A new sediment transport formulation was developed for the GSSHA model. The formulation can simulate erosion, transport, and deposition of any number, size, and specific gravity of particles in a continuous manner. This allows the deposition and erosion of varying materials to be tracked throughout a simulation period that can be days, weeks, months or years long. Additional enhancements include detachment by raindrop impact, flow detachment limits, and the addition of the Engelund-Hansen (1967) and stream power soil transport equations. The revised model shows good results for predicting sediment runoff volumes over an extended 3-month period. When tested on events much larger than any events in the calibration event, the accuracy of the model predictions did not deteriorate. When compared to the results from previous analysis (Ogden and Helig, 2001) the model is superior to the previous formulation in relation to three substantial concerns: 1) the ability to simulate small events, 2) the propagation of errors in discharge, and 3) the ability to simulate large events outside the range of calibration. The addition of detachment limits as well as other improvements to the formulation represents a clear improvement over previous versions of the model.

ADDITIONAL INFORMATION: This technical note was prepared by Dr. Charles W. Downer, Research Hydraulic Engineer, Coastal and Hydraulics Laboratory, U.S. Army Engineer Research and Development Center. The study was conducted as an activity of the Regional Sediment Management work unit of the System-Wide Water Resources Program (SWWRP). For information on SWWRP, please consult https://swwrp.usace.army.mil/ or contact the Program Manager, Dr. Steven L. Ashby at Steven.L.Ashby@usace.army.mil. This technical note should be cited as follows:

https://swwrp.usace.army.mil/

REFERENCES


**NOTE:** The contents of this technical note are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such products.